Bruchrechnung: Lösungen

1. $$5 \cdot \frac{7}{12} + 1 \cdot \frac{41}{72} + 2 \cdot \frac{17}{24} + 9 \cdot \frac{5}{9} = \frac{5 \cdot 7 \cdot 6 + 1 \cdot 41 + 2 \cdot 17 \cdot 3 + 9 \cdot 5 \cdot 8}{72} = \frac{713}{72}.$$

2. (a) $$\frac{b+5c-a}{6} - \frac{3a-7b+6c}{4} + \frac{4a-5b+7c}{3} = \frac{2(b+5c-a) - 3(3a-7b+6c) + 4(4a-5b+7c)}{12} = \frac{5a+3b+20c}{12}.$$

(b) $$\frac{1}{a+1} + \frac{4}{3a+2} - \frac{3}{a+1} = \frac{4}{3a+2} - \frac{2}{a+1} = \frac{4(a+1) - 2(3a+2)}{(3a+2)(a+1)} = \frac{-2a}{(3a+2)(a+1)}.$$

(c) $$\left(\frac{a}{3b} + \frac{3b}{a}\right) \cdot 3ab = a^2 + 9b^2.$$

(d) $$\left(\frac{a}{2b} - \frac{2b}{a}\right) : \frac{a}{a+2b} = \frac{a^2-4b^2}{2ab} \cdot \frac{a+2b}{a} = \frac{(a^2-4b^2)(a+2b)}{2a^2b} = \frac{(a-2b)(a+2b)^2}{2a^2b}.$$

(e) $$1 - \frac{1}{1 - a\frac{1}{1-\frac{b}{a}}} = 1 - \frac{1}{1 - a\frac{a}{a-b}} = 1 - \frac{a-b}{a-b-a^2} = \frac{a-b-a^2}{a-b-a^2} - \frac{a-b}{a-b-a^2}$$
$$= \frac{-a^2}{a-b-a^2} = \frac{a^2}{a^2-a+b}.$$

(f) $$\frac{\frac{a}{a-b} - \frac{b}{a+b}}{\frac{a}{a+b} + \frac{b}{a-b}} = \frac{a(a+b) - b(a-b)}{a(a-b) + b(a+b)} = \frac{a^2+b^2}{a^2+b^2} = 1.$$

3. (a) $$\frac{10!}{8!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} = 9 \cdot 10 = 90.$$

(b) $$\frac{13!}{7! \cdot 8!} = \frac{9 \cdot 10 \cdot 11 \cdot 12 \cdot 13}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} = \frac{3 \cdot 11 \cdot 13}{2 \cdot 7} = \frac{429}{14}.$$

(c) $$\frac{1}{6!} + \frac{2}{7!} + \frac{3}{8!} = \frac{7 \cdot 8}{8!} + \frac{2 \cdot 8}{8!} + \frac{3}{8!} = \frac{75}{8!} = \frac{3 \cdot 5 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8} = \frac{5}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 8} = \frac{5}{2688}.$$

(d) $$\frac{6! + 7!}{5! + 8!} = \frac{6! \cdot (1+7)}{5! \cdot (1 + 6 \cdot 7 \cdot 8)} = \frac{6 \cdot (1+7)}{1 + 6 \cdot 7 \cdot 8} = \frac{48}{337}.$$